Convergence of Self-Adaptive Evolutionary Algorithms

Evolutionary algorithms (EAs) are a heuristic global search strategy that has proven effective on a wide range of application domains. Specialized EAs have been developed to solve problems with continuous design parameters, and in particular Evolution Programming and Evolutionary Strategies (ES) have been used since the 1960s for applications with many local minima, and nonsmooth or noisy objective functions. For example, these EAs have been applied to applications like production planning for DOE's Pantex facility (see below). Despite their popularity, the mathematical convergence properties of these EAs are essentially unknown.

We have developed a new convergence theory for Evolutionary Strategies that provides mathematical confidence that these methods robustly converge to optimal solutions. Although it is widely recognized that EAs need to adapt the step-length parameters used for mutation, this work provides the first proof of convergence of a *standard* EA that performs such adaptation. We have analyzed the (1,λ)-ES, which uses *implicit self-adaptation*, the most popular technique for controlling step-lengths. This EA generates λ new points per iteration and keeps the best point; the step-lengths used to generate new points are co-evolved with the design parameters. Our analysis uses super-martingale theory to prove that the EA converges on separable, unimodal multi-dimensional problems.

This work will appear in the IEEE Transactions on Evolutionary Algorithms. A reviewer of this result wrote "... I consider this an outstanding contribution, providing a long-awaited result in the theory of continuous EAs." These mathematical insights allow decision makers to apply this EA with greater confidence in their analysis. Thus these methods can be effectively and robustly applied to maximize productivity, minimize risk, minimize cost, maximize reliability in computational models.

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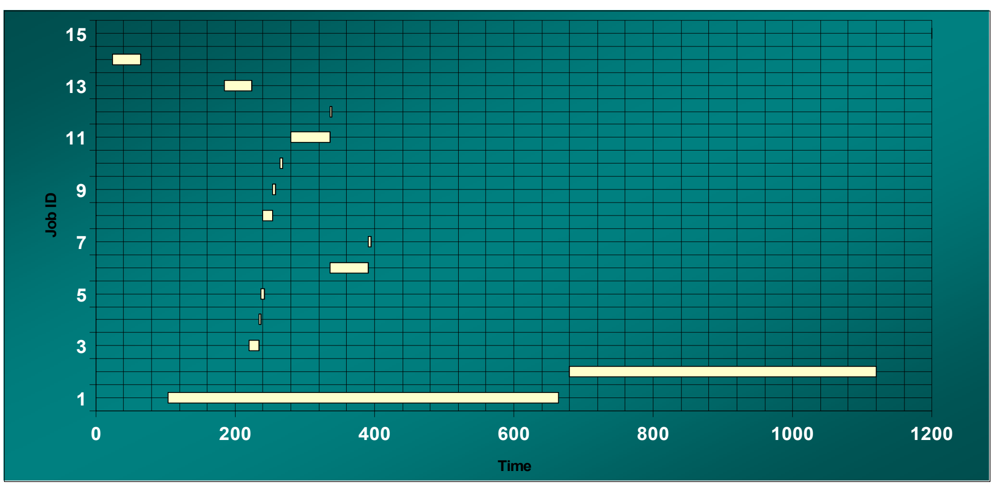

Figure 1: Illustration of a planning schedule developed for DOE's Pantex facility: Evolutionary algorithms have been applied to select job start times so as to minimize the number of additional staff and facility resources required to complete the schedule within a given time period.